CALCULUS

Ron Larson
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Solutions, Interactivity, Videos, & Tutorial Help at LarsonCalculus.com
### Basic Differentiation Rules

1. \( \frac{d}{dx}[cu] = cu' \)
2. \( \frac{d}{dx}[u \pm v] = u' \pm v' \)
3. \( \frac{d}{dx}[uv] = u'v + vu' \)
4. \( \frac{d}{dx} \left[ \frac{u}{v} \right] = \frac{vu' - uv'}{v^2} \)
5. \( \frac{d}{dx}[c] = 0 \)
6. \( \frac{d}{dx}[u^n] = nu^{n-1}u' \)
7. \( \frac{d}{dx}[x] = 1 \)
8. \( \frac{d}{dx} \left[ \frac{1}{u} \right] = \frac{-u'}{u^2} \)
9. \( \frac{d}{dx}[\ln u] = \frac{u'}{u} \)
10. \( \frac{d}{dx}[e^u] = e^u u' \)
11. \( \frac{d}{dx} \left[ (\ln a)u \right] = \frac{u'}{u} \)
12. \( \frac{d}{dx}[a^u] = (\ln a)a^u u' \)
13. \( \frac{d}{dx}[\sin u] = (\cos u)u' \)
14. \( \frac{d}{dx}[\cos u] = -(\sin u)u' \)
15. \( \frac{d}{dx}[\tan u] = (sec^2 u)u' \)
16. \( \frac{d}{dx}[\cot u] = -(csc^2 u)u' \)
17. \( \frac{d}{dx}[\sec u] = (\sec u \tan u)u' \)
18. \( \frac{d}{dx}[csc u] = -(csc u \cot u)u' \)
19. \( \frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}} \)
20. \( \frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}} \)
21. \( \frac{d}{dx}[\arccot u] = \frac{-u'}{1 + u^2} \)
22. \( \frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2} \)
23. \( \frac{d}{dx}[\arccsc u] = \frac{-u'}{|u|\sqrt{u^2 - 1}} \)
24. \( \frac{d}{dx}[\arcsec u] = \frac{u'}{|u|\sqrt{u^2 - 1}} \)
25. \( \frac{d}{dx}[\sinh u] = (\cosh u)u' \)
26. \( \frac{d}{dx}[\cosh u] = (\sinh u)u' \)
27. \( \frac{d}{dx}[\tanh u] = (\operatorname{sech} u)u' \)
28. \( \frac{d}{dx}[\coth u] = -(\operatorname{csch}^2 u)u' \)
29. \( \frac{d}{dx}[\operatorname{sech} u] = -(\operatorname{csch} u \tanh u)u' \)
30. \( \frac{d}{dx}[\operatorname{csch} u] = -(\operatorname{csch} u \coth u)u' \)
31. \( \frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2 + 1}} \)
32. \( \frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2 - 1}} \)
33. \( \frac{d}{dx}[\sec^{-1} u] = \frac{u'}{|u|\sqrt{u^2 - 1}} \)
34. \( \frac{d}{dx}[\coth^{-1} u] = \frac{u'}{u^2 - 1} \)

### Basic Integration Formulas

1. \( \int kf(u) \, du = k \int f(u) \, du \)
2. \( \int [f(u) \pm g(u)] \, du = \int f(u) \, du \pm \int g(u) \, du \)
3. \( \int du = u + C \)
4. \( \int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1 \)
5. \( \int a^u \, du = \left( \frac{1}{\ln a} \right) a^u + C \)
6. \( \int e^u \, du = e^u + C \)
7. \( \int \cos u \, du = \sin u + C \)
8. \( \int \sin u \, du = -\cos u + C \)
9. \( \int \cot u \, du = \ln |\sin u| + C \)
10. \( \int \tan u \, du = -\ln |\cos u| + C \)
11. \( \int \csc u \, du = -\ln |\csc u + \cot u| + C \)
12. \( \int \sec u \, du = \ln |\sec u + \tan u| + C \)
13. \( \int \csc^2 u \, du = -\cot u + C \)
14. \( \int \sec^2 u \, du = \tan u + C \)
15. \( \int \csc u \cot u \, du = -\csc u + C \)
16. \( \int \sec u \tan u \, du = \sec u + C \)
17. \( \int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C \)
18. \( \int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \arccsc \frac{|u|}{a} + C \)
19. \( \int \frac{du}{u \sqrt{a^2 - u^2}} = \frac{1}{a} \arcsin \frac{u}{a} + C \)
20. \( \int \frac{du}{u \sqrt{u^2 - 1}} = \frac{1}{a} \arctan \frac{u}{a} + C \)
Definition of the Six Trigonometric Functions

Right triangle definitions, where \( 0 < \theta < \pi/2 \).

- \( \sin \theta = \frac{\text{opp}}{\text{hyp}} \)
- \( \csc \theta = \frac{\text{hyp}}{\text{opp}} \)
- \( \cos \theta = \frac{\text{adj}}{\text{hyp}} \)
- \( \sec \theta = \frac{\text{hyp}}{\text{adj}} \)
- \( \tan \theta = \frac{\text{opp}}{\text{adj}} \)
- \( \cot \theta = \frac{\text{adj}}{\text{opp}} \)

Circular function definitions, where \( \theta \) is any angle.

- \( \sin \theta = \frac{y}{r} \)
- \( \csc \theta = \frac{r}{y} \)
- \( \cos \theta = \frac{x}{r} \)
- \( \sec \theta = \frac{r}{x} \)
- \( \tan \theta = \frac{y}{x} \)
- \( \cot \theta = \frac{x}{y} \)

Reciprocal Identities

- \( \sin x = \frac{1}{\csc x} \)
- \( \csc x = \frac{1}{\sin x} \)
- \( \sec x = \frac{1}{\cos x} \)
- \( \cot x = \frac{1}{\tan x} \)

Quotient Identities

- \( \tan x = \frac{\sin x}{\cos x} \)
- \( \cot x = \frac{\cos x}{\sin x} \)

Pythagorean Identities

- \( \sin^2 x + \cos^2 x = 1 \)
- \( 1 + \tan^2 x = \sec^2 x \)
- \( 1 + \cot^2 x = \csc^2 x \)

Cofunction Identities

- \( \sin \left( \frac{\pi}{2} - x \right) = \cos x \)
- \( \cos \left( \frac{\pi}{2} - x \right) = \sin x \)
- \( \sec \left( \frac{\pi}{2} - x \right) = \csc x \)
- \( \csc \left( \frac{\pi}{2} - x \right) = \sec x \)
- \( \tan \left( \frac{\pi}{2} - x \right) = \cot x \)
- \( \cot \left( \frac{\pi}{2} - x \right) = \tan x \)

Even/Odd Identities

- \( \sin(-x) = -\sin x \)
- \( \cos(-x) = \cos x \)
- \( \csc(-x) = -\csc x \)
- \( \tan(-x) = -\tan x \)
- \( \sec(-x) = \sec x \)
- \( \cot(-x) = -\cot x \)

Sum and Difference Formulas

- \( \sin(u \pm v) = \sin u \cos v \pm \cos u \sin v \)
- \( \cos(u \pm v) = \cos u \cos v \mp \sin u \sin v \)
- \( \tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} \)

Double-Angle Formulas

- \( \sin 2u = 2 \sin u \cos u \)
- \( \cos 2u = \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u \)
- \( \tan 2u = \frac{2 \tan u}{1 - \tan^2 u} \)

Power-Reducing Formulas

- \( \sin^2 u = \frac{1 - \cos 2u}{2} \)
- \( \cos^2 u = \frac{1 + \cos 2u}{2} \)
- \( \tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u} \)

Sum-to-Product Formulas

- \( \sin u + \sin v = 2 \sin \left( \frac{u + v}{2} \right) \cos \left( \frac{u - v}{2} \right) \)
- \( \sin u - \sin v = 2 \cos \left( \frac{u + v}{2} \right) \sin \left( \frac{u - v}{2} \right) \)
- \( \cos u + \cos v = 2 \cos \left( \frac{u + v}{2} \right) \cos \left( \frac{u - v}{2} \right) \)
- \( \cos u - \cos v = -2 \sin \left( \frac{u + v}{2} \right) \sin \left( \frac{u - v}{2} \right) \)

Product-to-Sum Formulas

- \( \sin u \sin v = \frac{1}{2} \left[ \cos(u - v) - \cos(u + v) \right] \)
- \( \cos u \cos v = \frac{1}{2} \left[ \cos(u - v) + \cos(u + v) \right] \)
- \( \sin u \cos v = \frac{1}{2} \left[ \sin(u + v) + \sin(u - v) \right] \)
- \( \cos u \sin v = \frac{1}{2} \left[ \sin(u + v) - \sin(u - v) \right] \)
AP*Edition
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Welcome to *Calculus*, Tenth Edition, AP* Edition. We are proud to present this new edition to you. As with all editions, we have been able to incorporate many useful comments from you, our user. For this edition, we have introduced some new features and revised others. You will still find what you expect – a pedagogically sound, mathematically precise, and comprehensive textbook.

We are pleased and excited to offer you something brand new with this edition – a companion website at LarsonCalculus.com. This site offers many resources that will help you as you study calculus. All of these resources are just a click away.

Our goal for every edition of this textbook is to provide you with the tools you need to master calculus. We hope that you find the changes in this edition, together with LarsonCalculus.com, will accomplish just that.

In each exercise set, be sure to notice the reference to CalcChat.com. At this free site, you can download a step-by-step solution to any odd-numbered exercise. Also, you can talk to a tutor, free of charge, during the hours posted at the site. Over the years, thousands of students have visited the site for help. We use all of this information to help guide each revision of the exercises and solutions.

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**New To This Edition**

**NEW** LarsonCalculus.com

This companion website offers multiple tools and resources to supplement your learning. Access to these features is free. Watch videos explaining concepts or proofs from the book, explore examples, view three-dimensional graphs, download articles from math journals and much more.

**NEW** AP* Exam Tips

Throughout the book, these tips offer important information about material that commonly appears on the AP* exam.

**NEW** Interactive Examples

Examples throughout the book are accompanied by Interactive Examples at LarsonCalculus.com. These interactive examples use Wolfram’s free CDF Player and allow you to explore calculus by manipulating functions or graphs, and observing the results.

**NEW** Proof Videos


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NEW How Do You See It?
The How Do You See It? feature in each section presents a real-life problem that you will solve by visual inspection using the concepts learned in the lesson. This exercise is excellent for classroom discussion or test preparation.

REVISED Remark
These hints and tips reinforce or expand upon concepts, help you learn how to study mathematics, caution you about common errors, address special cases, or show alternative or additional steps to a solution of an example.

REVISED Exercise Sets
The exercise sets have been carefully and extensively examined to ensure they are rigorous and relevant and include all topics our users have suggested. The exercises have been reorganized and titled so you can better see the connections between examples and exercises. Multi-step, real-life exercises reinforce problem-solving skills and mastery of concepts by giving students the opportunity to apply the concepts in real-life situations.

Table of Content Changes
Appendix A (Proofs of Selected Theorems) now appears in video format at LarsonCalculus.com.

The proofs also appear in text form at CengageBrain.com.

Trusted Features
AP® Review Questions
Modeled after the types of questions you will encounter on the AP® exam, these questions provide additional opportunities for practice and review.

Applications
Carefully chosen applied exercises and examples are included throughout to address the question, “When will I use this?” These applications are pulled from diverse sources, such as current events, world data, industry trends, and more, and relate to a wide range of interests. Understanding where calculus is (or can be) used promotes fuller understanding of the material.

Writing about Concepts
Writing exercises at the end of each section are designed to test your understanding of basic concepts in each section, encouraging you to verbalize and write answers and promote technical communication skills that will be invaluable in your future careers.
Theorems
Theorems provide the conceptual framework for calculus. Theorems are clearly stated and separated from the rest of the text by boxes for quick visual reference. Key proofs often follow the theorem and can be found at LarsonCalculus.com.

Definitions
As with theorems, definitions are clearly stated using precise, formal wording and are separated from the text by boxes for quick visual reference.

Explorations
Explorations provide unique challenges to study concepts that have not yet been formally covered in the text. They allow you to learn by discovery and introduce topics related to ones presently being studied. Exploring topics in this way encourages you to think outside the box.

Historical Notes and Biographies
Historical Notes provide you with background information on the foundations of calculus. The Biographies introduce you to the people who created and contributed to calculus.

Technology
Throughout the book, technology boxes show you how to use technology to solve problems and explore concepts of calculus. These tips also point out some pitfalls of using technology.

Section Projects
Projects appear in selected sections and encourage you to explore applications related to the topics you are studying. They provide an interesting and engaging way for you and other students to work and investigate ideas collaboratively.

Putnam Exam Challenges
Putnam Exam questions appear in selected sections. These actual Putnam Exam questions will challenge you and push the limits of your understanding of calculus.

Definition of Definite Integral
If \( f \) is defined on the closed interval \([a, b]\) and the limit of Riemann sums over partitions \( \Delta \)

\[
\lim_{|\Delta| \to 0} \sum_{i=1}^{n} f(c_i) \Delta x_i
\]

exists (as described above), then \( f \) is said to be integrable on \([a, b]\) and the limit is denoted by

\[
\lim_{|\Delta| \to 0} \sum_{i=1}^{n} f(c_i) \Delta x_i = \int_{a}^{b} f(x) \, dx.
\]

The limit is called the definite integral of \( f \) from \( a \) to \( b \). The number \( a \) is the lower limit of integration, and the number \( b \) is the upper limit of integration.

St. Louis Arch
The Gateway Arch in St. Louis, Missouri, was constructed using the hyperbolic cosine function. The equation used for construction was

\[
y = 693.8597 - 68.7672 \cosh 0.0100333x,
\]

\[-299.2239 \leq x \leq 299.2239\]

where \( x \) and \( y \) are measured in feet. Cross sections of the arch are equilateral triangles, and \((x, y)\) traces the path of the centers of mass of the cross-sectional triangles. For each value of \( x \), the area of the cross-sectional triangle is

\[
A = 125.1406 \cosh 0.0100333x.
\]

(Source: Owner’s Manual for the Gateway Arch, Saint Louis, MO, by William Thayer)

(a) How high above the ground is the center of the highest triangle? (At ground level, \( y = 0 \).)

(b) What is the height of the arch? (Hint: For an equilateral triangle, \( A = \sqrt{3}c^2 \), where \( c \) is one-half the base of the triangle, and the center of mass of the triangle is located at two-thirds the height of the triangle.)

(c) How wide is the arch at ground level?
Additional Resources

Student Resources

- **Student Solutions Manual for Calculus of a Single Variable**

- **Student Solutions Manual for Multivariable Calculus**
  (Chapters 11–16 of *Calculus*): ISBN 1-285-08575-2

  These manuals contain worked-out solutions for all odd-numbered exercises. Ask your sales representative about available discounts.

- **Fast Track to a 5 ISBN 1-285-06326-0**—This test preparation manual provides valuable test-taking strategies, review, and full-length practice exams. Keyed to *Calculus*, it helps students efficiently and effectively prepare for the AP exam. Ask your sales representative about available discounts. *The Fast Track to a 5* may be purchased with the text or separately.

  ![CourseMate](www.webassign.net)

  CourseMate is a perfect study tool for bringing concepts to life with interactive learning, study, and exam preparation tools that support the printed textbook. CourseMate includes: an interactive eBook, videos, quizzes, flashcards, and more! Ask your sales representative about available discounts.

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  (Chapters P–6 of *Calculus*): ISBN 1-285-08576-0

- **Complete Solutions Manual for Calculus of a Single Variable, Volume 2**
  (Chapters 7–10 of *Calculus*): ISBN 1-285-08577-9

- **Complete Solutions Manual for Multivariable Calculus**
  (Chapters 11–16 of *Calculus*): ISBN 1-285-08580-9

  The **Complete Solutions Manuals** contain worked-out solutions to all exercises in the text.

- **Solution Builder** (www.cengage.com/solutionbuilder)—This online teacher database offers complete worked-out solutions to all exercises in the text, allowing you to create customized, secure solutions printouts (in PDF format) matched exactly to the problems you assign in class.
• **PowerLecture (ISBN 1-285-09445-X)**—This comprehensive DVD includes resources such as an electronic version of the Teacher’s Resource Guide, complete pre-built PowerPoint® lectures, all art from the text in both jpeg and PowerPoint formats, ExamView® algorithmic computerized testing software, JoinIn™ content for audience response systems (clickers), and a link to Solution Builder.

• **ExamView Computerized Testing**— Create, deliver, and customize tests in print and online formats with ExamView®, an easy-to-use assessment and tutorial software. ExamView for *Calculus*, 10e contains hundreds of algorithmic multiple-choice and short answer test items. ExamView® is available on the PowerLecture DVD.

• **Teacher’s Resource Guide (ISBN 1-285-09445-X)**—This robust manual contains an abundance of resources keyed to the textbook by chapter and section, including chapter summaries and teaching strategies. An electronic version of the Teacher’s Resource Guide is available on the PowerLecture DVD.

**CourseMate**

CourseMate is a perfect study tool for students, and requires no set up from you. CourseMate brings course concepts to life with interactive learning, study, and exam preparation tools that support the printed textbook. CourseMate for *Calculus*, 10e includes: an interactive eBook, videos, quizzes, flashcards, and more! For teachers, CourseMate includes Engagement Tracker, a first-of-its kind tool that monitors student engagement. Ask your sales representative about available discounts.
We would like to thank the many people who have helped us at various stages of Calculus over the last 39 years. Their encouragement, criticisms, and suggestions have been invaluable.

Reviewers of the Tenth Edition
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Reviewers of Previous Editions
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On a personal level, we are grateful to our wives, Deanna Gilbert Larson and Consuelo Edwards, for their love, patience, and support. Also, a special note of thanks goes out to R. Scott O’Neil.

If you have suggestions for improving this text, please feel free to write to us. Over the years we have received many useful comments from both instructors and students, and we value these very much.

Ron Larson
Bruce Edwards
## Your Course. Your Way.

### Calculus Textbook Options

The traditional calculus course is available in a variety of textbook configurations to address the different ways instructors teach—and students take—their classes.

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<th>TOPICS COVERED</th>
<th>APPROACH</th>
<th>3-semester</th>
<th>Single Variable Only</th>
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<tr>
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<td>Calculus Early Transcendental Functions 5e</td>
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<td>Integrated coverage</td>
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AB Topics

Functions, Graphs, and Limits

(A) Analysis of Graphs
(i) Interplay between geometric and analytic information. 1.2–1.5, 3.5
(ii) Use of calculus to predict and explain the observed local and global behavior of a function. 3.1 - 3.6

(B) Limits of Functions (including one-sided limits)
(i) An intuitive understanding of the limiting process. 1.2–1.4
(ii) Calculating limits using algebra. 1.3–1.5, 3.5
(iii) Estimating limits from graphs or tables of data. 1.2, 1.5

(C) Asymptotic and Unbounded Behavior
(i) Understanding asymptotes in terms of graphical behavior. 1.5, 3.5
(ii) Describing asymptotic behavior in terms of limits involving infinity. 1.5, 3.5
(iii) Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth). 3.5, 5.1, 5.5, 8.7

(D) Continuity as a Property of Functions
(i) An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.) 1.4
(ii) Understanding continuity in terms of limits. 1.4, 1.5
(iii) Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem). 1.4, 3.1

Derivatives

(A) Concept of the Derivative
(i) Derivative presented graphically, numerically, and analytically. 2.1
(ii) Derivative interpreted as an instantaneous rate of change. 2.1, 2.2, 2.6, 6.2
(iii) Derivative defined as the limit of the difference quotient. 2.1
(iv) Relationship between differentiability and continuity. 2.1

(B) Derivative at a Point
(i) Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents. 2.1, 2.2, 6.1
(ii) Tangent line to a curve at a point and local linear approximation. 2.1–2.5, 6.1
(iii) Instantaneous rate of change as the limit of average rate of change. 2.2
(iv) Approximate rate of change from graphs and tables of values. 2.2–2.4

(C) Derivative as a Function
(i) Corresponding characteristics of graphs of \( f \) and \( f' \). 2.1, 3.1–3.6
(ii) Relationship between the increasing and decreasing behavior of \( f \) and the sign of \( f' \). 3.3
(iii) The Mean Value Theorem and its geometric interpretation. 3.2
(iv) Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa. 2.6, 3.7, 6.2

(D) Second Derivatives
(i) Corresponding characteristics of graphs of \( f, f', \) and \( f'' \). 3.4
(ii) Relationship between the concavity of \( f \) and the sign of \( f'' \). 3.4
(iii) Points of inflection as places where concavity changes. 3.4
**Correlation of AP Course Topics to Calculus**

**Applications of Derivatives**

1. Analysis of curves, including the notions of monotonicity and concavity. 3.1–3.6
2. Optimization, both absolute (global) and relative (local) extrema. 3.1–3.6
3. Modeling rates of change, including related rates problems. 2.6
4. Use of implicit differentiation to find the derivative of an inverse function. 5.3
5. Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration. 2.3, 3.2
6. Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations. 4.1, 5.1, 6.1

**Computation of Derivatives**

1. Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions. 2.2, 2.3, 5.1, 5.3–5.6
2. Derivative rules for sums, products, and quotients of functions. 2.2, 2.3
3. Chain rule and implicit differentiation. 2.4, 2.5

**Integrals**

**Interpretations and Properties of Definite Integrals**

1. Definite integral as a limit of Riemann sums. 4.3
2. Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:
   \[ \int_a^b f''(x) \, dx = f(b) - f(a) \]
3. Basic properties of definite integrals (examples include additivity and linearity). 4.3

**Applications of Integrals**

1. Use of integrals to model physical, biological, or economic situations. 4.4, 5.5, 6.2, 6.3, 7.4, 7.5
2. Use of the method of setting up an approximating Riemann sum and representing its limit as a definite integral. 4.3, 7.1, 7.2
3. Finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change. 4.2–4.6, 7.1, 7.2

**Fundamental Theorem of Calculus**

1. Use of the Fundamental Theorem to evaluate definite integrals. 4.4
2. Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined. 4.5

**Techniques of Antidifferentiation**

1. Antiderivatives following directly from derivatives of basic functions. 4.1, 4.4, 5.2, 5.4
2. Antiderivatives by substitution of variables (including change of limits for definite integrals). 4.5

**Applications of Antidifferentiation**

1. Finding specific antiderivatives using initial conditions, including applications to motion along a line. 4.1, 4.4, 6.1, 6.2
2. Solving separable differential equations and using them in modeling (including the study of the equation \( y' = ky \) and exponential growth). 6.1–6.3

**Numerical Approximations to Definite Integrals**

1. Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values. 4.2, 4.3, 4.6
**BC Only Topics**

*The AP BC exam includes all AB topics in addition to the following topics. Please note that the Chapter 12 vector topics are also covered in Appendix G: AP* BC Vector Topics (Web).

### Functions, Graphs, and Limits

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<th>(A) Parametric, Polar, and Vector Functions</th>
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### Derivatives

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<td>(ii) Numerical solution of differential equations using Euler’s method. 6.1</td>
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<tr>
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### Integrals

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<th>(A) Applications of Integrals</th>
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<td>(iii) Finding accumulated change (including a region bounded by polar curves) and the length of a curve (including a curve given in parametric form). 7.4, 10.3, 10.5</td>
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<tr>
<th>(B) Techniques of Antidifferentiation</th>
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<td>(ii) Improper integrals (as limits of definite integrals). 8.8</td>
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<th>(C) Applications of Antidifferentiation</th>
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### Polynomial Approximations and Series

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<td>(iii) Use of technology to explore convergence and divergence. 9.2</td>
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<th>(B) Series of Constants</th>
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<td>(vii) Comparing series to test for convergence or divergence. 9.4</td>
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### (C) Taylor Series

| (i)  | Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve). | 9.7 |
| (ii) | Maclaurin series and the general Taylor series centered at \( x = a \). | 9.7–9.10 |
| (iii) | Maclaurin series for the functions \( e^x, \sin x, \cos x, \text{ and } \frac{1}{1 - x} \). | 9.7–9.10 |
| (iv)  | Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series. | 9.8–9.10 |
| (v)   | Functions defined by power series. | 9.8–9.10 |
| (vi)  | Radius and interval of convergence of power series. | 9.8 |
| (vii) | Lagrange error bound for Taylor polynomials. | 9.7 |
Preparing for the AP* Calculus Examination

By: Sharon Cade, Mathematics Consultant; Rhea Caldwell, Providence Day School, Charlotte, North Carolina; and Jeff Lucia, Providence Day School, Charlotte, North Carolina

Advanced Placement can be exhilarating. Whether you are taking an AP course at your school or you are working on the AP curriculum independently, the stage is set for a great intellectual experience.

But sometime in the spring, when the examination begins to loom on a very real horizon, Advanced Placement can seem intimidating. It is a normal feeling to be nervous about the test; you are in good company.

The best way to deal with an AP examination is to master it, not let it master you. You should think of this examination as a way to show off how much calculus you know. Attitude does help. But, no matter what type of math student you are, there is still a lot you can do to prepare for the exam. Focused review and practice time will help you master the examination so that you can walk in with confidence and earn a great score.

What's in the Calculus Textbook that will help you prepare

As you work through the textbook, there are some things that you should do to get the most out of the text.

• Brush up on your precalculus skills by reviewing Appendix C (Web). Success in calculus comes not only from understanding the calculus but being able to do the mechanics, so the skills covered in review materials are key.

• Visit the LarsonCalculus.com website. The site contains a variety of tools and resources that will help you master critical calculus concepts.

• As you read the sections,
  * take time reading the examples,
  * do the conceptual exercises so that you can check your understanding of the material, and
  * notice that the author begins each set of exercises with the basic concepts and skills and proceeds to more challenging problems and applications; be sure to try some of each level.

• After finishing a chapter,
  * work through the Review Exercises and the P.S. Problem Solving Exercises and
  * complete the AP* Review Questions to practice your test-taking skills.

In calculus, reading the text is an essential part of the learning curve and will, in the end, save you time in understanding and mastering the material. There is more to the study of calculus than just being able to do some mathematics; you must understand the concepts and how they fit together. You will also learn to broaden your thinking as well as think logically if you allow yourself to try to see mathematics in a new way, not just as a set of algorithms.
How to get the most out of your Calculus class

• Know your advanced algebra skills:
  • Linear equations
  • Quadratics (factoring)
  • Functions (transformations, piecewise, odd, even, domain, range)
  • Polynomials (zeros, end behavior)
  • Exponential and logarithmic curves
  • Rational and radical equations
  • Direct, inverse relations
  • Conics (for BC)

• Know your trigonometry:
  • Unit circle (0, 30, 45, 60, and 90 degrees and equivalent radian measures)
  • Symmetry around the unit circle
  • Basic identities

• Know basic sequences and series and when to apply which sequence or series formula.

• Have some knowledge of vectors (BC only).

• Know your calculator:
  • Plot the graph of a function in an appropriate window.
  • Find the zeros of a function (solving the equation numerically).
  • Know how to evaluate a function at a specific value of \( x \): on the graph, 2nd calc, value; on the home screen, \( y_1(\text{value}) \).
  • Graph your function and analyze it as a comparison to the algebra you do.

Remember that you have to analyze what the calculator gives you. For example, know how to tell when the graph has a “hole:” \( y = \frac{x - 2}{x^2 - 4} \) and look for the value of \( y \) at \( x = 2 \).

Setting up a review schedule

The AP Calculus courses are concerned with developing a student’s understanding of the concepts while providing experiences with its methods and applications. Both the AB and BC courses require a depth of understanding. If you have been doing your homework steadily and keeping up with the course work, you are in good shape. Organize your notes, homework, and handouts from class by topic. For example, keep a set of notes on precalculus topics (no longer tested on the AP exam but essential to your success with calculus), limits, derivative rules and applications, integral rules and applications, series and sequences techniques and methods, and major theorems. If you can summarize the main information in a few pages, by topic, you will find reviewing much easier. Refer to these materials as you begin to prepare for the exam. Use your textbook to get more detail as needed.

AP Information before the examination

In February

• Make sure that you are registered to take the test. Some schools take care of the paperwork and handle the fees for their AP students, but check with your teacher or the AP coordinator to make sure that you are registered. This is especially important if you have a documented disability and need test accommodations. If you are studying AP independently, or if your school does not have an AP coordinator, call AP Services at the College Board at (609) 771-7300 or (888) 225-5427 (toll-free in the United States and Canada). You can also email apexams@info.collegeboard.org for the name of the local AP coordinator, who will help you through the registration process. It is suggested that you call AP Services or contact your AP coordinator by March 1.
• Check on the eligibility of your calculator. Go online to http://www.collegeboard.com/student/testing/ap/calculus_ab/calc.html?calcab#list early enough so that if you need a different calculator, you will have time to get one and become familiar with it.

By Mid-March
• Begin your review process; set a schedule for yourself that you can follow.

Night before
• Put all of your materials in one place.
• Relax and get a good night’s rest (this alone could improve your score because you will be able to think more clearly throughout the test).

Things to have on test day
• Approved graphing calculator with fresh batteries (you may have a second calculator as a backup, but it must also be a graphing calculator). The calculator must not have a typewriter-style (QWERTY) keyboard, nor can it be a non-graphing scientific calculator. Calculator memories are not cleared for the exam.

AP* Exam Tip
Be sure your calculator is set in radian mode (pi radians, or approximately three radians, is half the circle; 3 degrees is an angle just barely above the x-axis).

• #2 pencils (at least 2) with good erasers.
• A watch (to monitor your pace, but turn off the alarm if it has one).
• A bottle of water and a snack (fruit or power bar).
• Social Security number (if you choose to include it on the forms).
• The College Board school code.
• Photo identification and the admissions ticket.
• Comfortable clothes and a sweatshirt or sweater in case the room is cold.

Schools may have your admissions ticket at the testing site; a photo identification may not be needed at your own school, but check with your AP coordinator prior to test day.

On the day of the examination, it is wise to eat a good breakfast. Studies show that students who eat a hot breakfast before testing get higher scores. Breakfast can give you the energy you need to power through the test and more. You will spend some time waiting while everyone is seated in the right room for the right test. That’s before the test has even begun. With a short break between Section I and Section II, the AP Calculus exam can last almost four hours.

Now go get a 5!
To do well on the AP Calculus examination, you should be able to:

- Understand and work with the connections between the graphical, numerical, analytical, and verbal representations of functions.
- Use derivatives to solve a variety of problems and understand the meaning of a derivative in terms of rate of change and local linearity.
- Use integrals to solve a variety of problems and understand the meaning of a definite integral in terms of the limit of Riemann sums as well as the net accumulation of change.
- A student also needs to:
  - Understand both parts of the Fundamental Theorem of Calculus.
  - Communicate mathematics in written sentences.
  - Appropriately model a physical situation.
  - Use technology correctly and efficiently.
  - Determine whether solutions are reasonable and understand them in terms of units of measurement, size, and so on.

It is important to realize that a student who is in AP Calculus is expected to have studied all of the prerequisite material. A student should have a mastery of functions and their properties and an understanding of algebra, graphs, and the language of domain, range, symmetry, periodicity, and so on. The student should also understand trigonometry and have a mastery of the basic values in the unit circle and the basic trigonometric identities.

**Exam Format**

The AP Calculus examination currently consists of two major sections, each of which has two parts. All sections test proficiency on a variety of topics.

**Multiple Choice:** Section I has two sets of multiple-choice questions. Part A has 28 questions with an allotted time of 55 minutes and does not allow the use of a calculator. Part B has 17 questions with an allotted time of 50 minutes and does contain some questions for which a graphing calculator is needed. The multiple choice section score is based on the number of questions answered correctly; no points will be deducted for incorrect answers and no points are awarded for unanswered questions.

**Free Response:** Section II has six free-response questions, and it is broken into two parts. Part A consists of two problems, in which some parts of the problems may require a graphing calculator. You will be allowed 30 minutes for Part A. Part B has four problems, in which a calculator is not permitted. You will be allowed 60 minutes for Part B. Although you may continue working on Part A problems during this 60-minute session, you may no longer use a calculator. Thus, when working on Part A, you must be sure to answer the questions requiring a calculator during that first 30-minute period.

The grade for the examination is equally weighted between the multiple-choice and free-response sections. You can possibly earn a 5 on the exam even if you miss an entire free-response question. Students taking the BC exam will also receive an AB subscore grade.

The free-response questions and solutions are published annually after the AP Reading is completed and can be found at apcentral.com.
General AP Test-Taking Strategies

Strategize the test question. Begin somewhere. Ask “What do I need?” and then “How do I get there?” Start with a clear definition; for example, for question 6 about continuity on the 2003 AB test, students needed to have a clear definition of continuity to answer the question fully.

• Know what the required tools on your calculator are and know how to access and use them:
  - Plot the graph of a function within a viewing window.
  - Find the zeros of a function (numerically solve equations).
  - Numerically calculate the derivative of a function.
  - Numerically calculate the value of a definite integral.

• Know the relationships between $f, f', \text{ and } f''$.

• Know your differentiation and integration rules.

• Underline key components of the questions.

• Treat units carefully.

• Set the calculator to at least THREE decimal places and properly use the store key for intermediate steps (if you round too soon, your final answer will not be correct to the requisite three decimal places). Although you only see three decimal places on the calculator, it actually retains the complete value in its memory; if you just write the three numbers down and then use them, you are rounding too soon. If you choose to write your answer with more than three decimal places, remember only the first three places are read as your answer—you may truncate or round.

Strategies for the Multiple-Choice Section

To Look or not to look at the answers: Often you are better off trying to work through the problem before you look at the answer choices. However, there are times when seeing the answer choices may help you determine a method of solution (see 1998 multiple-choice test question 17 where only in the answer do you see that you have to compare $f(1)$ to $f'(1)$ to $f''(1)$).

Read the question carefully: Pressured for time, many students make the mistake of reading the questions too quickly or merely skimming them. By reading a question carefully, you may already have some idea about the correct answer. Careful reading is especially important in EXCEPT questions. After you solve the problem and have a solution, reread the question to be sure your answer you solved for actually answers the question. For example, you may have solved for where the maximum occurred (the $x$-value), but the question actually asks for the maximum value of $f$ (the $y$-value), and thus you need one more step to complete the problem.

Eliminate any answer you know is wrong: You can write on the multiple-choice questions in the test book. As you read through the responses, draw a line through any answer you know is wrong. Do as much scratch work as is necessary in the exam book, but be sure to mark your solution choice on the answer sheet in the corresponding oval.

Read all of the possible answers, then choose the most accurate response: AP examinations are written to test your precise knowledge of a subject. Some of the responses may be partially correct, but there will only be one response that is completely true.

Be careful of absolute responses. These answers often include the words always or never. They could be correct, but you should try to think of counterexamples to disprove them.
Skip tough questions: Skip them in the first go-through, but be sure that you mark them in the margin so you can come back to them later if you possibly can. Make sure you skip those questions on your answer sheet, too.

There is no penalty for guessing: Thus, at the end, try to narrow down your choices by eliminating answers you think are incorrect and make an educated guess.

Additional Thoughts

• The exact numerical answer may not be among the choices given. You will have to choose the solution that best approximates the exact numerical value.

• The domain of a function $f$ is assumed to be the set of all real numbers $x$, where $f(x)$ is a real number, unless specified otherwise.

• $f^{-1}$ or the prefix arc- indicates the inverse of a trigonometric function ($\cos^{-1} x = \arccos x$).

Types of Multiple-Choice Questions

All kinds of topics will be covered in the multiple-choice section; your skills and vocabulary will be tested as well as your ability to do multi-step problem solving. Phrases like average value, the definition of continuity, extremum (relative and absolute), the definition of a derivative in its two forms, differential equations, graphical interpretations, and slope fields are just a sampling of terms that identify the kinds of problems you will see. Read through this text and do the practice problems to familiarize yourself with the way the questions are framed.

Multiple-choice questions will be formatted in two basic ways. You will find classic questions where there are just five answer choices. This is the most common type of problem; it requires you to read the question and select the correct answer. Strategies for solving this type of problem include

• reading the question carefully,

• eliminating known wrong answers,

• solving the problem and then interpreting your solution to fit the question, and

• on occasion, testing each solution choice to see which one is correct.

There will also be problems that could be called “list” and “group,” where you may be asked “Which of the following is true about $g$?” You will be given choices such as I, II, and III, and the multiple-choice answers might appear as:

(a) None  
(b) I only  
(c) II only  
(d) I and II only  
(e) I, II, and III

This kind of problem requires a clear understanding of a concept or definition. To approach this kind of question,

• eliminate known wrong answers,

• recall necessary theorems or definitions to help you interpret the question, and

• reread the problem to check your solution’s accuracy.
Strategies for the Free-Response Section

ALL work needs to be shown IN the test booklet, not on the question sheets. The readers must read only solutions in the test booklet itself. Be sure to write the solutions on the correct pages (answer for question 1 on the designated page). It is easy to write a solution on the wrong page. If this happens, make a note of it on the correct page and the reader will read with you; do not take the time to erase and redo the problem.

- Scan all of the questions in the section you are working in. First solve the problems that you think you can do easily. You can mark and come back to the harder problems later. During Part A (the calculator section with only 2 problems), be sure to tackle any parts of questions that requires the use of the calculator first, because after the 30 minutes, although you can continue to work on the problems, you will not be allowed to use your calculator.

- Show all of your work. Partial credit will be awarded for problems if the correct work is shown, even if the answer is not present or is incorrect. Although not required, it can be helpful to the reader if you circle your final answer.

- Cross out incorrect answers with an “X” rather than spending time erasing. Crossed-out or erased work will not be graded. However, don’t cross out or erase work unless you have replaced it. Let the reader see what you tried; it may be worth some points.

- Be clear, neat, and organized in your work. If a reader cannot clearly understand your work, you may not receive full credit.

- Some free-response questions have several parts, such as a, b, c, and d. Attempt to solve each part. Even if your answer to part “a” is incorrect, you still may be awarded points for the remaining parts of the question if the work is correct for those parts. Remember, the answers may not depend on an earlier response and that is why it is important to try each part. If you work with your incorrect answer (as long as it is reasonably derived) from a previous part, the reader will read with you in a later part, checking your numerical answer on the basis of your incorrect input.

- Units are important in your answer. Keeping track of your units throughout calculations and performing unit cancellations, where possible, will help guide you to your answer. Points will be deducted for missing or incorrect units in the answer if the question asked that units be given.

- Don’t just write equations or numbers in hopes of finding the correct answer. Extraneous or incorrect information could lead to a lower score. Don’t make up work that is trivial, but do try the problem and the reader will read with you.

- You do not need to answer the questions in order, but be sure the answer is entered in the correct section.

- When you use a table or a graph from one section (part a) in another part of the problem (part c, for example), be sure to refer to it in some way—state your use of it or draw an arrow back to it. If you inadvertently put a response in the wrong part of the problem, note it clearly to the reader.

- Show all your work.
  - Clearly label any functions [if the problem uses g(x), don’t call it f(x)].
  - Label your sign charts accurately, for example, f’ or f” for the derivative tests. However, these by themselves do not count as a justification. No credit is given for sign charts, but they can help develop your analysis and subsequent answer to the question.
  - Label all graphs with appropriate notation including numeric intervals (by 1s or 10s, for example) and the names for the x- and y-axes (like distance and time).
  - Label all tables or other objects that you use to show your work.
  - Show standard mathematical (noncalculator) notation. For example, you must show the integral as \( \int_1^3 (x + 2) \, dx \), not as fnInt \( (x + 2, x, 1, 3) \).
Remember: You are not required to simplify your answer; you can save both time and the opportunity to make an error by leaving an answer in an unsimplified form. For example, \(y - 2.3 = -6(x + 5.4)\) is an appropriate equation of a tangent line; there is no need to simplify it to slope-intercept form.

- Decimals require an accuracy of three decimal places in the solution. Thus be sure to understand how to carry (store in your calculator) the intermediate steps of a problem until you round to three decimal places at the end of the problem. If you do multiple calculations and each calculation is rounded to three decimal places prior to the next calculation, your final solution will not have the required accuracy. The third digit in the final solution can be either rounded or truncated.

**Scoring for Free-Response Questions**

The free-response sections are graded on a scale of 0–9 with a dash (−) given for no math on the page. The chief reader is ultimately responsible not only for working through the solution and alternate solutions for each problem, but also for assigning points on a 9-point scale to each problem. This varies from problem to problem, based on how many parts are in the problem as well as the complexity of the problem.

For example, in a problem that asks for units, units are generally assigned 1 point for the whole problem. In other words, if you do units correctly in part a, but incorrectly in part c, you would not be awarded the 1 point for units.

If a problem requires an explanation, or reasoning, it generally earns 1–2 points.

In a typical area and volume problem, the integral is often worth 1 point and the answer is worth an additional 1 point. Sometimes the limits of integration are also worth 1 point. Thus, it is important for you to at least get started on the problem because often some points are earned on the setup, even if the solution is not there or is incorrect.

A “bald” answer is one that has no supporting work or documentation, like “yes” or just a number. A bald answer is seldom awarded a point.

To learn calculus and best prepare for the examination, read the text, take risks, ask questions, and look for the connections between the algebraic, numeric, and graphical approaches to similar problems. As much as possible, graph every problem to enhance your understanding of the concepts involved.